The Sailboat Problem

by

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MATERIALS

1. The sailboat (Fig. 1): a block of wood 24 cm long, 7 cm wide, and 2 cm thick. Glued at the centre of this is another wooden block, 20 cm long, 4.5 cm wide, and 2.5 cm thick. Four roller-skate wheels are attached to this glued block. Slots of 1 cm depth and of width such that one can easily mount and remove the cardboard in these slots are cut on the top of the first block one parallel to the wheel axles but at the centre of the block, another perpendicular to the axles along the keel but at the centre of the block (call it the keel slot), a third at about a 20° angle to the keel slot but at the centre, and the fourth at about a 45° angle to the keel slot but again at the centre.
2. The sail: stiff cardboard of area 929 cm^2 (one square foot) to be placed in the slots, at various angular positions.
3. An electric fan.

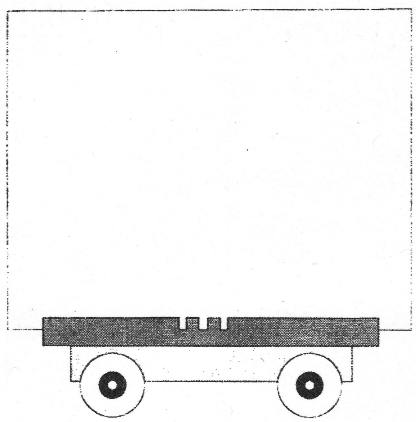


Figure 1

DISCUSSION AND DEMONSTRATIONS

The sailboat provides one of the most interesting illustrations of vector resolution. Some of the many questions raised are:

1. Suppose you are sailing directly downwind with your sails full, in a 30 km/hr wind. What maximum speed would you hope to attain?
2. You are sailing downwind and you pull your sail in so that it no longer makes a 90° angle with the keel of the boat. What will this tactic do to the speed of the boat?
3. Keeping the angle of the sail relative to the boat the same as in the previous question, suppose you now direct your boat so that it sails directly across the wind, rather than directly with the wind. Will you sail faster or slower than before?
4. Can a sailboat travel against the wind?

Consider first the case of a sailboat sailing downwind (Fig. 2). The force of the wind impact against the sail accelerates the boat. Even if the drag of water and all other resistance forces are negligible, the maximum speed of the boat is the wind speed. This is because the wind will not make an impact against the sail if the boat is moving as fast as the wind. The sail will simply sag. If there is no unbalanced force, then there is no acceleration. The force vector decreases as the boat travels faster. The force vector is minimum when the boat travels as fast as the wind. Hence the boat, when driven only by the wind, cannot exceed the wind speed.

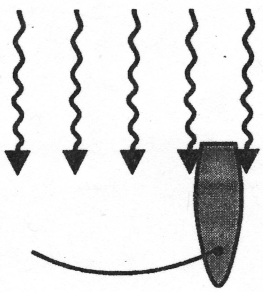
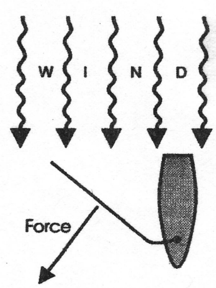
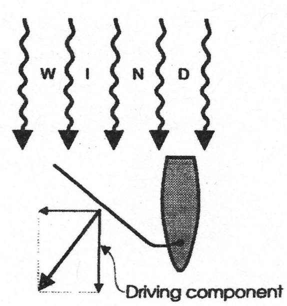


Figure 2: A sailboat sailing downwind

If the sail is oriented at an angle as shown in Fig. 3, the boat will move forward, but with less acceleration. The reason for this can be stated in two different, but equivalent, ways:



1. The force on the sail is less because the sail does not intercept as much wind as in this angular position.



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2. The direction of the wind impact force on the sail is not in the direction of the boat's motion, but is perpendicular to the surface of the sail. Generally speaking, when any fluid (liquid or gas) interacts with a smooth surface, the force of the interaction is perpendicular to the smooth surface.

The boat does not move in the same direction as the perpendicular force on the sail, but is constrained to move in a forward (or backward) direction by a deep, finlike keel beneath the water. In our demonstration, the four wheels determine this direction. The component of the force perpendicular to the keel is a useless force that tends to tip the boat over or move it sideways. Again, maximum speed of the boat can be no greater than the wind speed. However, because the acceleration is less, the time required to attain the maximum speed is greater.

Keeping the angle of the sail relative to the boat the same as in Fig. 3, suppose now you direct your boat so that it sails directly across the wind (Fig. 4), rather than directly with the wind. Will you sail faster or slower than before? The answer is faster.

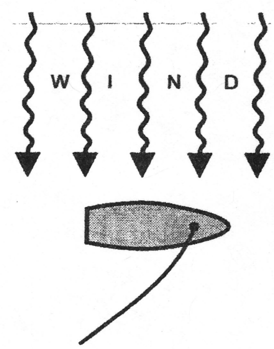

Figure 4

As before, the force vector perpendicular to the surface of the sail can be broken into components, one along the direction in which the boat can move, which drives the boat, and the other which is perpendicular to the boat's motion and is almost useless. (This transverse force is not entirely useless the generation of a small angle of "heel" increases the waterline length, and because of complex hydrodynamic effects, increases the boat speed somewhat.) Now, if the principal force vector in this case were not greater than before, the speed of the boat would be the same. But the force vector is greater. The reason is that the sail does not catch up with the wind speed so it will not eventually sag like before. Even when the boat is travelling as fast as the wind, there is an impact of wind against the sail. This drives the boat even faster, so it can sail faster than the wind in this position. It reaches its terminal speed when the 'relative wind' the resultant of the 'natural' wind and the 'artificial' wind due to the boat's motion blows along the sail without making impact (Fig. 5).

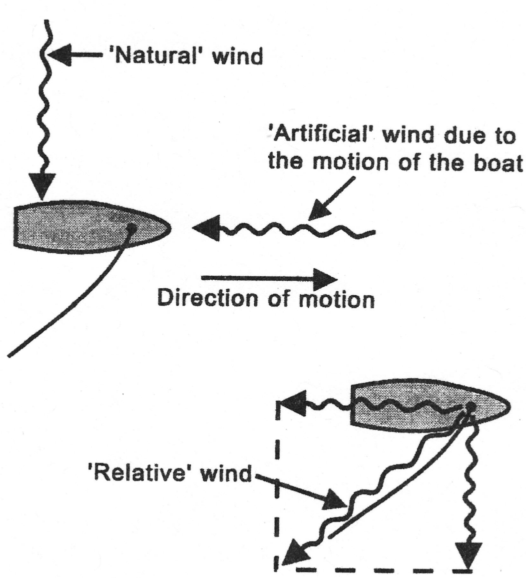

Figure 5: When the angle of the relative wind is the same as the sail angle, the wind impact is zero

It is very interesting to note that, if the wind speed is doubled, the impact against the sail is more than doubled. This is because in one second twice as much air strikes the sail and at twice the speed, so twice the mass moving twice as fast produces four times the force.

As strange as it may seem, maximum speed is attained by cutting into (against) the wind, that is, by angling the sailboat in a direction upwind. Although a sailboat cannot sail directly upwind, it can reach a destination upwind by angling back and forth in a zigzag fashion. This is called **tacking** (Fig. 6).

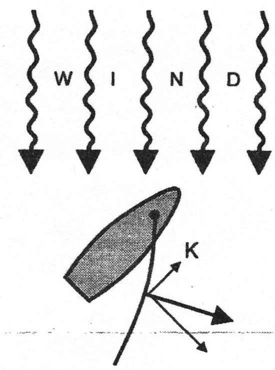

Figure 6

Component **K** will push the boat along in a forward direction, angling into the wind. In this situation, the boat can sail faster than the speed of the wind. This is because, as the boat travels faster, the impact of wind is increased. The boat reaches its terminal speed when opposing forces cancel the force of wind impact. The opposing forces consist mainly of water resistance against the hull of the boat. The hulls of racing boats are shaped to minimize this resistive force, which is the principal deterrent to high speeds.

Because of its minimal drag on the surface, an ice boat can go up to an estimated five times the speed of the wind. The official iceboat speed record, 230 km/h, was set by an old-fashioned stern-steerer in 1938, but the unofficial record is claimed by a giant yacht which covered 1.9 km in 25 seconds, reaching about 274 km/h.